

Mathematical Dissertations

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January 9th 2019

1 Algebra

It seems like a global epidemic which, together with climate change, lead us to the tragedy. A kind of fashion in which the people come to the belief that the only thing needed to make things happen is to wish them.. Or, worse still, when it comes to a politician, promise them.

For these people when a paleontologist claims that a dinosaur fed on something in particular, they think, is like a divination that lacks sustenance, since that said paleontologist, obviously, was not there, to witness how those creatures feed.

It is therefore that I raise the need to disseminate, much more strongly, that one of the most important features of algebra is its capability to deduced information using the environment in which it is obtained. And, if you study algebra since the basic levels of education, is not to find the value of x , but to experience how the values of the unknowns can be found through the use of logic. I.e., the algebra is a discipline that helps us see how logic can find information with infinite precision.

Thus, for example in the equation

$$x^2 - 10x - y + 25 = 0$$

x and y are unknown values, or values to be defined, and can represent an infinite number of situations, but there present, even not playing the leading role, are still other symbols and numbers that constitute the environment and limit the possibilities of the variables.

Anyone with higher level mathematical training recognizes, that y value can not be arbitrary, negative values are prohibited for y (Figure 1), of course, would be naive to think that someone with elementary algebraic training recognize this feature, but what is indispensable, is to recognize that there is no magic and that if one can infer a value is due to the knowledge and application of the arithmetic rules and that these may not apply to discretion, as well as the natural law, applied to all and above all.

So, when a mathematician find or set the value of a variable, it is not doing magic, is not guessing and much less is finding a wished value.

Unfortunately, politicians are unaware of that, or that hate experts, come with suspicion as they seem to find the values of the unknowns and abuse the ignorance making promises that simply cannot be met.

2 Functions and Notation

In social media “problems” like the following are seen frequently:

$$\begin{array}{rclcl} 5 & \times & 5 & = & 7 \\ 7 & \times & 7 & = & 13 \\ 12 & \times & 12 & = & ? \end{array}$$

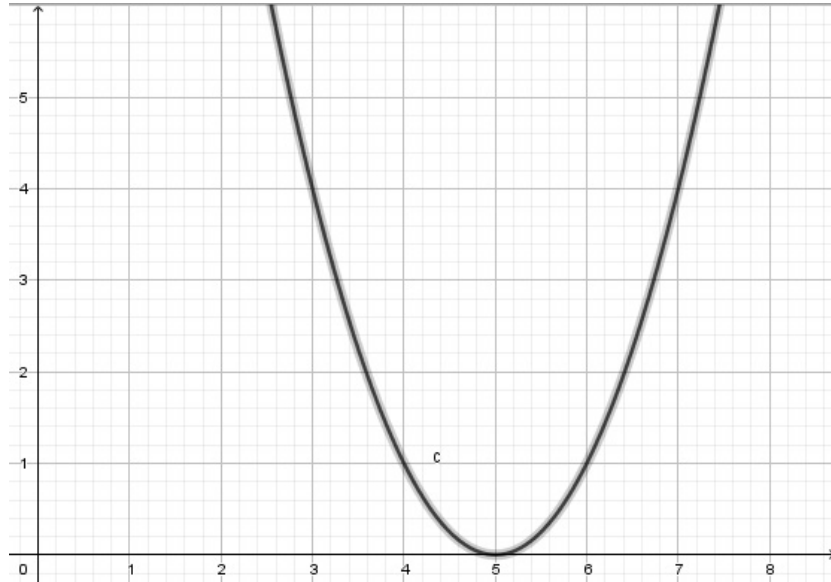


Figure 1: Parabola characterized by the coefficients $a=1$, $b=-10$, $c=25$.

is useless to mention the number of inaccuracies or mistakes there captured, intention is also of dubious moral quality, but we can analyze the reactions of people who try to “solve” such “problems”.

If expressions were formal, those would imply a redefinition of the numerical basis, i.e., the multiplication of the symbol 5 (which could be representing any quantity) equals the symbol 7 (which can be representing any quantity, e.g. 25) that’s what goes through my mind when I see these curious exercises, however, for the vast majority of the people, it means, to look for a function that relates the left side of the equations (in the normal decimal system) with the number on the right, to express it formally what everyone sees is:

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ such that:

$$\begin{aligned} f(5^2) &= 7 \\ f(7^2) &= 13 \end{aligned}$$

Define $f(x)$ and calculate $f(12)$

Written that way my anxiety level is decreased exponentially, but the amount of people willing to suggest a response decreases at the same rate.

It is curious as all that amount of mathematical symbols as well as formality designed to avoid ambiguities usually scares, if not that mentally blocks, the vast majority of the population.

It is clear that there is no limit to the number of functions that you can set as the solution that meet the conditions 49 with 13 and 25 with 7. Draws attention, equally, the limited amount of functions that have a symbol.

That is, if the function that you have in mind is algebraic simply express operations that have to be made using the variable, but in case of a non-algebraic process, to my mind only occur two functions that have earned not only one but several symbols to express themselves. Those are the derivative and the integral. And just for the pleasure of writing, below the examples:

$$\frac{df}{dx}; \quad f'; \quad f''; \quad \dot{x}; \quad \ddot{x}; \quad \nabla f; \quad \nabla^2 f$$

$$\oint_{\mathcal{C}} f(z) dz; \quad \int_{-\infty}^{+\infty} f(x) dx; \quad \int_{\mathbf{R}^2} f(x, y) dx dy$$

Now, these functions only apply on algebraic expressions, even when they reach abuses wishing to express that the derivative is an instant rate of change and the integral an infinite sum of infinitesimal pieces over things that are not algebraic expression.

The previous reasoning is just to highlight that almost all of the people who issued a response that involves a process of deduction (not divination) mentally defines a function that adds the digits that make up the numbers. So the relation between 49 and 13 is that the latest is the sum of 4 and 9. The fact that the most popular answer this implies that it is also the simplest function a brain can find or which requires less energy. Thus, the most common answer is $f(12) = 9$.

For me the easiest function to find is an algebraic one due to find a function that sums all the digits in a number i can just think in an algorithm and maybe write a piece of software. My answer for this “riddle” is to find a line that touches the points (49,13) and (25,7) and evaluate at desired value. Then

$$f(x) = \frac{1}{4}x + \frac{3}{4}$$

Finally $f(144) = \frac{147}{4}$